

Practice Calculus Test without Trig

The problems here are similar to those on the practice test. Slight changes have been made.

1. What is the domain of the function $f(x) = \sqrt{3x-1}$? Express the answer in interval notation.

The domain is the set of values x can take. Unless there is reason to believe otherwise, it is usually assumed that $f(x)$ is real. For $f(x)$ to be real, the expression under the radical sign must be 0 or greater. Express that as an inequality and solve:

$$3x - 1 \geq 0$$

$$3x \geq 1$$

$$x \geq 1/3$$

In interval notation, that's $[1/3, \infty)$.

2. What is the range of $f(x) = x^2 - 9$? Express the answer in interval notation.

The range is the set of values y can take. What values can y take here? Note that x^2 is always positive or 0. As the absolute value of x gets large, $f(x)$ will increase without limit, so the x^2 part of $f(x)$ will be 0 at least and possibly much more. The value 9 will be subtracted from x^2 . When $x = 0$, $f(x) = -9$. When $x \neq 0$, $f(x) > -9$, with no limit to how high it can go as the absolute value of x increases.

In interval notation, that's $[-9, \infty)$.

3. Find the composition of the functions $f(x) = e^{2x}$ and $g(x) = 2x + 4$, $(f \circ g)(x)$.

To find $(f \circ g)(x)$, perform the function $f(x)$ on $g(x)$, treating $g(x)$ as if it were x . In other words, $(f \circ g)(x) = f(g(x))$.

$$(f \circ g)(x) = f(2x + 4) = e^{2(2x+4)} = e^{(4x+8)}$$

4. Simplify $(2a^{-3})^4$.

Consider these rules for exponents:

Rule	Example	How It Applies Here
$(bc)^n = b^n c^n$	$(2 \cdot 3)^2 = (2 \cdot 3)(2 \cdot 3) = 2^2 \cdot 3^2$	$(2a^{-3})^4 = 2^4(a^{-3})^4$
$(b^c)^d = b^{cd}$	$(2^3)^2 = (2 \cdot 2 \cdot 2)^2 = 8^2 = 64 = 2^6$	$2^4(a^{-3})^4 = 16a^{-12}$
$b^{-c} = 1/b^c$	$2^{-1} = 1/2$	$16a^{-12} = 16/a^{12}$

The final answer is $16/a^{12}$.

5. Simplify $x^{2a+3}x^{4a}$.

To multiply a base taken to a power by the same base taken to a power (either the same power or a different one), add the exponents. The product is the same base taken to the **sum** of the powers.

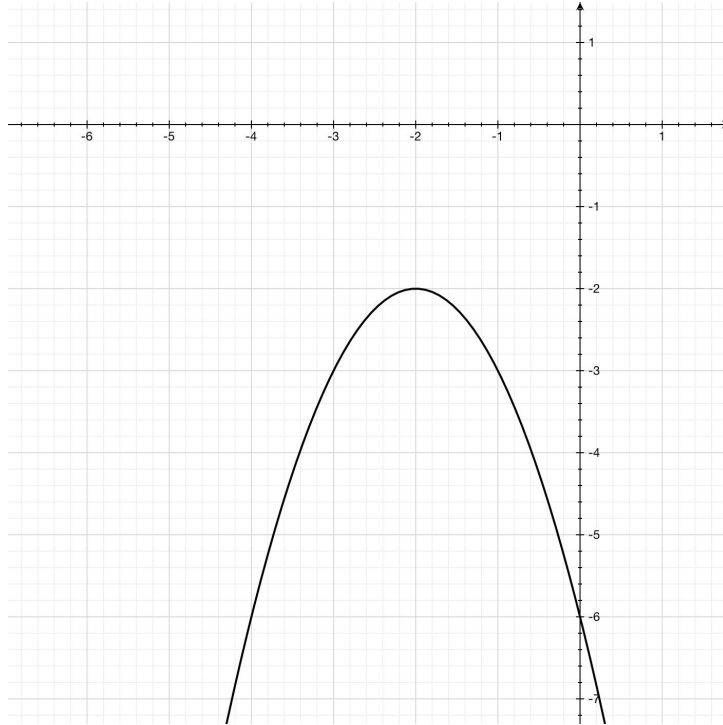
People want to add exponents here, but that's not right. Consider a simple example: $3^2 \cdot 3^4 = (3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3)$ Count up all the 3's and you see that that's 3^6 . And $6 = 2 + 4$, the sum of the exponents.

It works the same with complicated algebraic expressions: Add the exponents.

$$2a + 3 + 4a = 6a + 3, \text{ so}$$

$$x^{2a+3}x^{4a} = x^{6a+3}$$

6. Write the equation of the parabola shown in the graph.



The equation of a parabola in vertex format, the format most useful for this problem, looks like $f(x) = a(x - h)^2 + k$, where (h, k) is the location of the vertex.

This parabola's vertex, as you can see, is at $(-2, -2)$. So its equation is $f(x) = a(x + 2)^2 - 2$. You still have to find a . To do that, substitute the coordinates of another point on the parabola into the equation you have. Any point on the parabola will do, but usually a point with an x -value one unit away from that of the vertex is easiest. Try $(-1, -3)$, a point you can see on the graph:

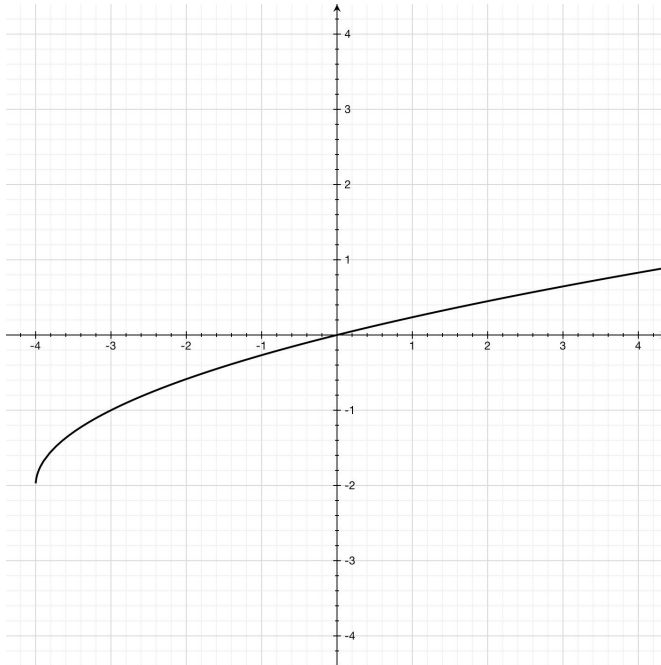
$$-3 = a(-1 + 2)^2 - 2$$

$$-1 = a(1)^2$$

$$-1 = a \text{ or } a = -1$$

Then $f(x) = -(x + 2)^2 - 2$ and that's the parabola's equation.

7. Write the equation of the square root function shown in the graph.



The equation you need will be in the form $f(x) = a\sqrt{x-h} + k$, where (h, k) are the coordinates of the vertex of what is, after all, the upper half of a rotated parabola. On this graph that is the left-hand endpoint. You can read its coordinates: $(-4, -2)$. So the graph's equation is $f(x) = a\sqrt{x+4} - 2$. To find a , substitute the coordinates of another point on the graph into the equation. The origin $(0, 0)$ looks like an easy point on the graph:

$$0 = a\sqrt{0+4} - 2$$

$$a\sqrt{4} = 2$$

$$2a = 2$$

$$a = 1$$

Finally, the equation is $f(x) = \sqrt{x+4} - 2$.

8. Given $f(x) = 7x^3 + 3$, find $f^{-1}(x)$.

The term $f^{-1}(x)$ means the inverse of $f(x)$. The inverse undoes the original function. It is the function you get if you reverse the roles of x and y in the original function.

So do that: reverse the roles of x and y in the original function:

$$y = 7x^3 + 3$$

$$\text{Reverse: } x = 7y^3 + 3$$

But you need to show this relation with y isolated on the left-hand side, so solve for y :

$$x = 7y^3 + 3$$

$$x - 3 = 7y^3$$

$$\frac{x-3}{7} = y^3$$

$$y = \sqrt[3]{\frac{x-3}{7}}$$

$$f^{-1}(x) = \sqrt[3]{\frac{x-3}{7}}$$

And that's the correct answer.

9. Graph the function

$$f(x) = \begin{cases} 3^x & \text{for } x \leq 1 \\ 4x - 1 & \text{for } x > 1 \end{cases}$$

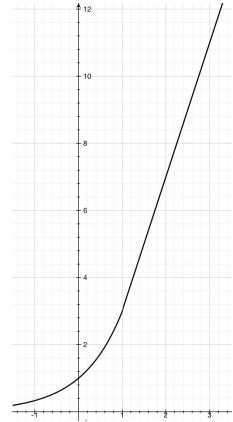
Find the y -values for several x -values in each piece of this piecewise-defined function:

x	$f(x)$
-1	1/3
0	1
1	3
Just a tiny bit >1	Just a tiny bit >3
2	7
3	11

The part on the left, for $x \leq 1$, is an exponential equation with a base that is greater than 1. The graph of such an equation is asymptotic to the x -axis as x

approaches negative infinity. As you go further to the right, the graph increases, um, exponentially. At the end of the left-hand piece, where $x = 1$, $3^x = 3$.

The other piece, which picks up just after $x = 1$, is a straight line with a positive slope of 4. If this piece did extend left all the way to $x = 1$, the two pieces would meet at the point $(1, 3)$.



It graphs up like this:

10. Given

$$f(x) = \begin{cases} \log(x-2) & \text{for } x \leq 4 \\ x^3 - 2 & \text{for } x > 4 \end{cases}$$

evaluate $f(3)$.

Note that only one piece of this function is relevant to the problem. You're asked to evaluate the function for an x -value less than 4, so consider only the piece for $x \leq 4$: $f(x) = \log(x-2)$.

$$y = \log(3-2)$$

$$y = \log(1)$$

$$y = 0$$

11. Given $f(x) = 5x + 2$, simplify

$$\frac{f(x+h)-f(x)}{h}$$

If $f(x) = 5x + 2$, then $f(x+h) = 5(x+h) + 2$. Then

$$\frac{f(x+h)-f(x)}{h} = \frac{5(x+h)+2-(5x+2)}{h} = \frac{5x+5h+2-5x-2}{h} = \frac{5h}{h} = 5$$

The answer is 5.

12. Given $f(x) = 3x^2 - 4x$, simplify $\frac{f(x+h)-f(x)}{h}$.

Note that:

$$f(x+h) = 3(x+h)^2 - 4(x+h)$$

$$= 3(x^2 + 2hx + h^2) - 4x - 4h$$

$$= 3x^2 + 6hx + 3h^2 - 4x - 4h$$

Then

$$\frac{f(x+h)-f(x)}{h} = \frac{3x^2+6hx+3h^2-4x-4h-(3x^2-4x)}{h}$$

$$= \frac{6hx+3h^2-4h}{h}$$

$$= 6x + 3h - 4$$

13. Rationalize the denominator.

$$\frac{2}{\sqrt{x}-4}$$

To rationalize the denominator, you need to multiply the expression by 1 in the form of the denominator's conjugate divided by itself.

$$\frac{2}{\sqrt{x}-4} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4} = \frac{2\sqrt{x}+8}{x-16}$$

14. Divide. Give your answer in $a + bi$ format.

$$\frac{2}{4-3i}$$

$\frac{2}{4-3i}$	
$\frac{2}{4-3i} \left(\frac{4+3i}{4+3i} \right)$	Multiply by 1 in the form of the denominator's conjugate divided by itself.
$= \frac{8+6i}{25}$	Carry out the multiplication.
$= \frac{8}{25} + \frac{6i}{25}$	Reformat into two terms.

15. Write as a single logarithm:

$$\log(4x + 1) - \log(x - 2) - 4\log x$$

$$4\log x = \log x^4$$

Power rule for logarithms.

$$\begin{aligned} &\log(4x+1) - \log(x-2) - 4\log x \\ &= \log(4x+1) - \log(x-2) - \log x^4 \end{aligned}$$

This step, while technically not part of making the expression into one term, is part of what is expected when you are asked to write an expression as a single expression.

$$= \log \frac{(4x+1)}{(x-2) \cdot x^4}$$

Quotient rule for logarithms.

16. Expand the expression $\ln\left(\frac{xy^2}{4z}\right)$.

$$\begin{aligned} \ln(xy^2) &= \ln x + \ln(y^2) \\ \ln(4z) &= \ln 4 + \ln z \end{aligned}$$

Product rule for logarithms

$$\ln(y^2) = 2\ln y$$

$$\ln(xy^2) = \ln x + 2\ln y$$

Power rule for logarithms

$$\ln\left(\frac{xy^2}{4z}\right) = \ln(xy^2) - \ln(4z)$$

Quotient rule for logarithms

$$\ln\left(\frac{xy^2}{4z}\right) = \ln x + 2\ln y - (\ln 4 + \ln z)$$

$$\ln\left(\frac{xy^2}{4z}\right) = \ln x + 2\ln y - \ln 4 - \ln z$$

Substitution. Notice that the two terms in the denominator are both treated the same way. (It is a common error to add the second denominator term rather than subtract it.)

17. Write $\sqrt[5]{x^4}$ without the radical sign.

To write a radical without a radical sign, use exponential notation: $\sqrt[n]{y} = y^{\frac{1}{n}}$.

Apply that to $\sqrt[5]{x^4}$ and you get $\sqrt[5]{x^4} = x^{\frac{4}{5}}$.

18. Solve the following equation for w .

$$v = \log_5(3w - 2)$$

$$v = \log_5(3w - 2)$$

$$5^v = 3w - 2$$

Basic logarithmic relationship: $y = \log_a x$ is equivalent to $x = a^y$.

$$w = \frac{5^v + 2}{3}$$

Solve for w .

19. Solve the equation for x :

$$\log_3 x + \log_3(x - 8) = 2$$

$$\log_3 x + \log_3(x - 8) = 2$$

$$\log_3[x(x - 8)] = 2$$

Product rule for logarithms.

$$\log_3(x^2 - 8x) = 2$$

$$3^2 = x^2 - 8x$$

Basic logarithmic relationship: $y = \log_a x$ is equivalent to $x = a^y$.

$$x^2 - 8x - 9 = 0$$

Rewrite the equation in quadratic format.

$$(x - 9)(x + 1) = 0$$

Factor.

$$\begin{array}{ll} x - 9 = 0 & x + 1 \\ x = 9 & x = -1 \end{array}$$

Solve.

Checking your work is always a good idea, but in logarithm problems it's especially important, because the domain of the logarithmic function is limited. If one of your answers falls outside the domain, that answer is *extraneous*, not a valid answer.

$$\log_3 x + \log_3(x - 8) = 2$$

Check the answer $x = 9$:

$$\log_3 9 + \log_3 (9 - 8) =$$

$$\log_3 9 + \log_3 1 =$$

$$2 + 0 = 2. \text{ Check.}$$

Check the answer $x = -1$:

$$\log_3 (-1) + \log_3 (-1 - 8) =$$

$$\log_3 (-1) + \log_3 (-9) =$$

But what is $\log_3 (-1)$ or $\log_3 (-9)$?

The domain of the log function includes only **positive** real numbers; an answer that requires you to take the log of a negative number is extraneous.