New 8/1/2018

NOVA COLLEGE-WIDE COURSE CONTENT SUMMARY
MTH 281 – INTRODUCTORY ABSTRACT ALGEBRA (3 CR.)

Course Description

Introduces groups, isomorphisms, fields, homomorphisms, rings, and integral domains. Applicable to some education licensure programs; not intended for STEM majors. Lecture 3 hours. Total 3 hours per week.

General Course Purpose

To provide an introduction to abstract mathematics and rigorous proof in the context of algebraic structures to students seeking endorsement to teach mathematics at the secondary level.

Course Prerequisites/Corequisites

Prerequisite: Completion of MTH 263, Calculus I with a grade of C or better or equivalent.

Course Objectives

Upon completing the course, the student will be able to:

- Introduction to Logic and Proof
    - Demonstrate the proof writing strategies of direct proof, indirect proof (proof of contrapositive), and proof by contradiction in the context of proving basic results about integers (e.g. "Prove that the product of two odd integers is odd.")
    - State and apply the Well Ordering Principle (for the naturals) and General Well Ordering Principle (for sets of integers bounded below) to conclude whether a given set is guaranteed a smallest element.
    - Write induction proofs in the context of proving basic results about integers
- Operations and Relations
    - State and apply the definition of an equivalence relation on a set and determine which properties (reflexive, symmetric, transitive) a defined relation on a given set passes or fails.
    - State and apply the definition of a partial order, and determine if a defined relation on a given set is anti-symmetric
    - Construct equivalence classes given a set and equivalence relation
    - Determine if a given operation on a set is well defined
    - Illustrate the construction of integers as equivalence classes of ordered pairs of natural numbers with defined operations of addition and multiplication
- Divisibility and Prime Numbers
    - State and apply the definition of divides and prove basic results about divisibility of integers (e.g. "if a|b and b|c, then a|c")
    - Given two integers a and b, apply the Division Algorithm to express a = bq + r, 0 <= r < b
    - Use the Euclidean Algorithm to find the greatest common divisor of a pair of integers
    - Obtain integer solutions to Diophantine equations and write the general form of the solution
    - Write the prime factorization of a given natural number
    - State and prove the Fundamental Theorem of Arithmetic
    - State and prove results about prime numbers
    - Apply the definition of the Euler phi-function to determine the number of numbers relatively prime to a given integer
    - State and apply Euler's Theorem and Fermat's Little Theorem
- Modular Arithmetic, Congruence, and an Introduction to Zm
    - State and apply the definition of congruence modulo m
    - State and prove fundamental properties of the congruence relation
    - Perform modular arithmetic on congruence classes of integers

- State and prove results about solutions to linear congruences, and apply them to determine solutions
- Solve systems of linear congruences
- Define Zm and its operations and perform arithmetic within Zm for a given m
- Prove that the operations on Zm satisfy the properties of commutativity and associativity of addition and multiplication, and the distributive property of multiplication over addition
- Solve linear equations within Zm for a given m

- Rings, Fields, and Integral Domains
  - State the definitions of ring, commutative ring, ring with unity, integral domain, and field
  - Given a set and two binary operations, determine which of the above structures it falls under by verifying algebraic properties (examples including integer/rational/real/complex numbers with addition and multiplication, Zm rings, and sets and operations in the abstract as defined by Cayley tables)
  - Determine if an element of a ring is a zero divisor, a unit, or neither
  - Perform algebraic operations in the complex field, including applying De Moivre's Theorem to compute powers and nth roots
- Polynomials
  - Perform algebraic operations on polynomials in Q[x] and Z[x] (rational and integer coefficients), and also in Zm[x] (coefficients in a Zm ring with arithmetic modulo m).
  - Perform long division of polynomials in F[x] (F a field, including Q, Z, C, and Zm, m prime) and express in the form of the Division Algorithm
  - Use the Euclidean algorithm to find the greatest common divisor of two polynomials in F[x]
  - State, prove, and apply the Remainder/Root Theorems for polynomials
  - State and prove the Unique Factorization Theorem for polynomials in F[x]
  - Determine if a polynomial is reducible in F[x] (apply relevant theorems such as Eisenstein's Criterion); if so, factor completely
  - State the Fundamental Theorem of Algebra, and display an understanding of the concepts underlying the proof
- Groups, Isomorphism, and Homomorphism
  - State the definitions of group and Abelian group, and state and prove additional basic properties of groups (e.g. (xy)^-1=y^-1x^-1)
  - Given a set and a binary operation, determine whether it is a group (and if Abelian) by verifying algebraic properties (examples including integer/rational/real/complex numbers with addition or multiplication, the Klein-4 group, and sets and operations in the abstract as defined by Cayley tables)
  - Construct Cayley tables for the groups Um formed from the units of Zm with the operation of multiplication and perform arithmetic in Um
  - Define the dihedral groups of symmetries of the triangle and the square and implement operations on elements within the groups
  - State and apply the definitions of subgroup, proper subgroup, and cyclic subgroup
  - Construct direct (Cartesian) product groups
  - State the definition of an isomorphism between two groups and be able to determine if one exists by identifying an operation preserving bijection
  - State the definition of a homomorphism between two groups and be able to determine if one exists by identifying an operation preserving map

Major Topics to be Included

a) Introduction to Logic and Proof
b) Operations and Relations
c) Divisibility and Prime Numbers
d) Modular Arithmetic, Congruence, and an Introduction to Zm
e) Rings, Fields, and Integral Domains
f) Polynomials
g) Groups, Isomorphism, and Homomorphism