Chapter 3 Homework Solutions

P: 1, 7, 9, 11, 13

**P1.** A car is driven 215 km west and then 85 km southwest. What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.

**Solution:**

Choose: North is "up" and "+"
   East is "+"

\[
\begin{align*}
D_{1x} &= -215 \text{ km} \quad \text{(points west)} \\
D_{1y} &= 0 \text{ km} \\
D_{2x} &= -D_2\cos(45^\circ) = -60.1 \text{ km} \quad \text{(points west)} \\
D_{2y} &= -D_2\sin(45^\circ) = -60.1 \text{ km} \quad \text{(points south)} \\
D_x &= D_{1x} + D_{2x} = -215 \text{ km} - 60.1 \text{ km} = -275.1 \text{ km} \\
D_y &= D_{1y} + D_{2y} = 0 - 60.1 \text{ km} = -60.1 \text{ km} \\
D &= \sqrt{D_x^2 + D_y^2} = \sqrt{(-275.1 \text{ km})^2 + (-60.1 \text{ km})^2} = 281.6 \text{ km} \\
\tan(\theta) &= \frac{D_y}{D_x} = \frac{-60.1 \text{ km}}{-275.1 \text{ km}} = 0.218 \Rightarrow \theta = 12.3^\circ \quad \text{(pointing south of west)}
\end{align*}
\]

**P7.** \(\mathbf{V}\) is a vector 14.3 units in magnitude and points at an angle of 34.8° above the negative x axis.
(a) Sketch this vector.
(b) Find \(V_x\) and \(V_y\).
(c) Use \(V_x\) and \(V_y\) to obtain (again) the magnitude and direction of \(\mathbf{V}\)
Solution:

(a)

(b) 
\[
\begin{align*}
V_x &= \cos(34.8^\circ) \Rightarrow V_x = V \cos(34.8^\circ) = 11.74 \text{ units} \\
V_y &= \sin(34.8^\circ) \Rightarrow V_y = V \sin(34.8^\circ) = 8.16 \text{ units}
\end{align*}
\]

(c) 
\[
\begin{align*}
V &= \sqrt{V_x^2 + V_y^2} = \sqrt{(11.74)^2 + (8.16)^2} = 14.3 \text{ units} \\
\tan(\theta) &= \frac{V_y}{V_x} = \frac{8.16}{11.74} = 0.695 \Rightarrow \theta = 34.8^\circ
\end{align*}
\]

P9. An airplane is traveling 735 km/h in a direction 41.5° west of north.
(a) Find the components of the velocity vector in the northerly and westerly directions.
(b) How far north and how far west has the plane traveled after 3.00 h?

Solution:

(a) 
\[
\begin{align*}
V_x &= \sin(41.5^\circ) \Rightarrow \\
V_x &= V \sin(41.5^\circ) = (735 \text{ km/h}) \sin(41.5^\circ) = 487.0 \frac{\text{km}}{\text{h}}
\end{align*}
\]
\[
\frac{V_y}{V} = \cos(41.5^\circ) \Rightarrow \\
V_y = V \cos(41.5^\circ) = (735 \text{ km/h}) \cos(41.5^\circ) = 550.5 \text{ km/h}
\]

(b) \( V = \text{const} \), \( \Rightarrow \) displacement = (velocity)(time)
\[
y = y_o + V_y t = 0 \text{ km} + (550.5 \text{ km/h}) \times (3 \text{ h}) = 1651.5 \text{ km}
\]
\[
x = x_o + V_x t = 0 \text{ km} + (487.0 \text{ km/h}) \times (3 \text{ h}) = 1461.0 \text{ km}
\]

**Fig. 1**

\[\text{P11. Determine the vector } \vec{A} - \vec{C} \text{, given the vectors } \vec{A} \text{ and } \vec{C} \text{ in the Fig.1}
\]

**Solution:**

\[\text{\vec{A} - \vec{C} is drawn in red on the diagram. It is a vector that points from the tip of } \vec{C} \text{ to the tip of } \vec{A}
\]

We can determine its magnitude and direction by adding the components of \( \vec{A} \) and \( \vec{C} \):
\[
\begin{align*}
\frac{A_x}{A} &= \cos(28.0^\circ) \Rightarrow A_x = (44.0)\cos(28.0^\circ) = 38.85 \\
\frac{A_y}{A} &= \sin(28.0^\circ) \Rightarrow A_y = (44.0)\sin(28.0^\circ) = 20.66
\end{align*}
\]
The magnitude of the vector then is:

\[ |\overrightarrow{A} - \overrightarrow{C}| = \sqrt{(38.85)^2 + (51.66)^2} = 64.6 \]

The direction is determined by the angle between the vector and the x axis:

\[ \tan(\theta) = \frac{(\overrightarrow{A} - \overrightarrow{C})_y}{(\overrightarrow{A} - \overrightarrow{C})_x} = \frac{51.66}{38.85} = 1.33 \Rightarrow \theta = 53.1^\circ \]

**P13.** For the vectors given in the Fig.1, determine

(a) \( \overrightarrow{A} - \overrightarrow{B} + \overrightarrow{C} \)

(b) \( \overrightarrow{A} + \overrightarrow{B} - \overrightarrow{C} \)

(c) \( \overrightarrow{C} - \overrightarrow{A} - \overrightarrow{B} \)

**Solution:**

We first calculate the components of the three vectors:

\[ A_x = (44.0)\cos(28.0^\circ) = 38.85 \]
\[ A_y = (44.0)\sin(28.0^\circ) = 20.66 \]

\[ B_x = -(26.5)\cos(56.0^\circ) = -14.82 \]
\[ B_y = (26.5)\sin(56.0^\circ) = 21.97 \]

\[ C_x = 0 \]
\[ C_y = -31.0 \]

(a) \( \overrightarrow{A} - \overrightarrow{B} + \overrightarrow{C} \) = ?

\[ \left( \overrightarrow{A} - \overrightarrow{B} + \overrightarrow{C} \right)_x = A_x - B_x + C_x = 38.85 - (-14.82) + (0) = 53.67 \]
\[ \left( \overrightarrow{A} - \overrightarrow{B} + \overrightarrow{C} \right)_y = A_y - B_y + C_y = 20.66 - (21.97) + (-31.0) = -32.37 \]

\[ |\overrightarrow{A} - \overrightarrow{B} + \overrightarrow{C}| = \sqrt{(53.67)^2 + (-32.37)^2} = 62.7 \]
\[ \tan(\theta) = \frac{-32.37}{53.67} = -0.603 \Rightarrow \theta = -31.1^\circ \]

The "−" in the angle means that the vector is below the x-axis.

(b) \( \overrightarrow{A} + \overrightarrow{B} - \overrightarrow{C} \) = ?
\[
\begin{align*}
(x + y - z) &= A_y + B_y - C_y = 20.66 + (21.97) - ( -31.0) = 73.63 \\
|A + B - C| &= \sqrt{(24.03)^2 + (73.63)^2} = 77.45 \\
\tan(\theta) &= \frac{73.63}{24.03} = 3.06 \Rightarrow \theta = 71.9^\circ
\end{align*}
\]

(c) \(-A - B + C\) = ?
Notice that \((A + B - C) = -(-A - B + C)\)

\[
\begin{align*}
(x - y + z) &= -A_x - B_x + C_x = -24.03 \\
(y - y + z) &= -A_y - B_y + C_y = -73.63
\end{align*}
\]

\[
|\mathbf{-A} - \mathbf{B} + \mathbf{C}| &= \sqrt{(-24.03)^2 + (-73.63)^2} = 77.45 \\
\tan(\theta) &= \frac{73.63}{24.03} = 3.06 \Rightarrow \theta = 71.9^\circ
\]

Notice that the angle turns out to be the same as (b). The x and y components, however, are negative meaning that the vector lies in III quadrant, while the vector in (b) has all positive components and lies in I quadrant.

P: 1, 19, 21, 31, 33, 37*, 39*