Chapter 09  Static and Equilibrium

Conditions for Equilibrium:

1. Sum of all forces is zero

\[ \sum \vec{F} = \vec{0} \]

2. Sum of all torque is zero

\[ \sum \tau = \vec{0} \]

Q3: How can we find the CG of meter stick?

Q9: Can the sum of the torque on an object be zero while the net force on the object is non zero?

Example 9-2  Chandelier cord tension

Calculate the tensions \( \vec{F}_A \) and \( \vec{F}_B \) in the two cords that are connected to the vertical cord supporting the 200-kg chandelier in Fig.9-4

\[
\begin{align*}
\text{Along } x: & \quad F_b - F_{ax} = 0 \\
\text{Along } y: & \quad F_{ay} - 1960 \text{ N} = 0 \\
\frac{F_{ax}}{F_b} &= \cos(60^\circ) \Rightarrow F_{ax} = F_a \cos(60^\circ) \\
\frac{F_{ay}}{F_b} &= \sin(60^\circ) \Rightarrow F_{ay} = F_a \sin(60^\circ) \\
F_b - F_a \cos(60^\circ) &= 0
\end{align*}
\]
\[ F_a \sin(60^\circ) - 1960 \text{ N} = 0 \]

\[ F_a = \frac{1960 \text{ N}}{\sin(60^\circ)} = 2.263 \times 10^3 \text{ N} \]

\[ F_b = (2.263 \times 10^3 \text{ N}) \cos(60^\circ) = 1.13 \times 10^3 \text{ N} \]

Example 9-4: **Balancing a seesaw**

A board of mass \( M = 2.0 \text{ kg} \) serves as a seesaw for two children, as shown in Fig. 9-7a. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, \( P \) (his center of gravity is 2.5 m from the pivot). At what distance \( x \) from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.

\[ \sum \vec{F} = \vec{0} \rightarrow \text{along } x : 0 \]

\[ \text{along } y : F_N - Mg - m_a g - m_b g = 0 \]

\[ \sum \tau = \vec{0} \rightarrow -m_b g(x) + m_a g(2.5 \text{ m}) = 0 \]

**Note:** \( F_N \) and \( Mg \) do not have torque (x=0)

\[ m_b g(x) = m_a g(2.5 \text{ m}) \]

\[ x = \frac{m_a g(2.5 \text{ m})}{m_b g} = \frac{30 \text{ kg} (2.5 \text{ m})}{25 \text{ kg}} = 3.0 \text{ m} \]

Example 9-6: **Hinged beam and cable**

A uniform beam, 2.20 m long with mass \( m = 25.0 \text{ kg} \), is mounted by a hinge on a wall as shown in Fig. 9-10. The beam is held in a horizontal position by a cable that makes an angle \( \theta = 30^\circ \) as shown. The beam supports a sign of mass \( M = 28.0 \text{ kg} \) suspended from its end. Determine the components of the force \( F \) that the hinge exerts on the beam, and the tension in the supporting cable.
\[ F_{Ty} = F_T \sin(30^\circ) \quad F_{Tx} = F_T \cos(30^\circ) \]

\[ \sum\overrightarrow{F} = 0 \]
\[ \sum F_x = F_{Hx} - F_{Tx} = 0 \]
\[ \sum F_y = F_{Ty} + F_{Hy} - mg - M g = 0 \]

\[ \sum \tau = 0 \]
\[ - M g L - mg \frac{L}{2} + F_{Ty} L = 0 \]
\[ F_{Ty} = \frac{M g L + mg L}{L} = (M + \frac{m}{2}) g = 397 \text{ N} \]
\[ F_T = \frac{F_{Ty}}{\sin(30^\circ)} = 794 \text{ N} \]

\[ F_{Hx} = F_{Tx} = 794 \text{ N} \cos(30^\circ) = 687.6 \text{ N} \]
\[ F_{Hy} = mg + M g - F_{Ty} = 123 \text{ N} \]
\[ F_H = \sqrt{(F_{Hx})^2 + (F_{Hy})^2} = 793 \text{ N} \]

<table>
<thead>
<tr>
<th>Stable Equilibrium:</th>
<th>The object returns to its equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unstable Equilibrium:</td>
<td>The object moves farther away</td>
</tr>
<tr>
<td>Neutral Equilibrium:</td>
<td>The object does not move</td>
</tr>
</tbody>
</table>
Elasticity and Stress

Hooke's Law

\[ F = k \Delta L \]

- \( F \) – applied force
- \( k \) – coefficient of proportionality, depends on the material
- \( \Delta L \) – change in length

Elastic Region → Plastic Region → Break Point

\[ \Delta L \propto \]

1. \( F \) – applied force
2. \( \frac{1}{A} \) – cross section
3. \( L \) – length

\[ \Delta L = \frac{1}{E} \frac{F}{A} L \]

- \( E \) – Young’s Modulus (elastic modulus)

Consequences:

1. Longer ropes stretch more
2. Thicker ropes stretch less

Elastic Modulus Shear Modulus Bulk modulus

"stretching" "compressing"

Table 9-1: values

Example: A force of 10^5 N is applied to an aluminum rod \((E = 70 \times 10^9 N/m^2)\) with a length of 2.0 m and a cross section of 40 cm^2. By how much does it stretch? If we cut the rod in half and tie the two parts in parallel next to each, and apply the same amount of force, by how much will they stretch?

\[
\begin{align*}
\Delta L &= \frac{1}{E} \frac{F}{A} L \\
\Delta L &= \frac{1}{E} \frac{F}{A} = \frac{1}{70 \times 10^9 N/m^2} \frac{1 \times 10^5 N}{40 \times 10^{-4} m^2} = 3.573 \times 10^{-3} = 0.36\% \\
\Delta L &= 7.1 \times 10^{-3} m = 7.1 \text{ mm} \\
\end{align*}
\]

\[
\begin{align*}
\Delta L' &= \frac{1}{E} \frac{F}{A} L' \\
\Delta L' &= \frac{1}{E} \frac{F}{A} = \frac{1}{70 \times 10^9 N/m^2} \frac{1 \times 10^5 N}{20 \times 10^{-4} m^2} = 1.786 \times 10^{-3} = 0.18\% \\
\Delta L' &= 1.8 \times 10^{-3} m = 1.8 \text{ mm} \\
\end{align*}
\]

\[
\begin{align*}
\frac{\Delta L'}{\Delta L} &= \frac{\frac{L'}{A}}{\frac{L}{A}} = \frac{L' A}{L A} = \frac{L' A}{L A} = \frac{1}{2} = \frac{1}{4}
\end{align*}
\]