Chapter 08 Homework solutions

P1. Express the following angles in radians: (a) 30°, (b) 57°, (c) 90°, (d) 360°, and (e) 420°. Give as numerical values and as fractions of \( \pi \).

Solution:
(a) \( (30^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = \frac{\pi}{6} \text{ rad} = 0.52 \text{ rad} \)
(b) \( (57^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = \frac{19\pi}{60} \text{ rad} = 0.99 \text{ rad} \)
(c) \( (90^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = \frac{\pi}{2} \text{ rad} = 1.57 \text{ rad} \)
(d) \( (360^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = 2\pi \text{ rad} = 6.28 \text{ rad} \)
(e) \( (420^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = \frac{7\pi}{3} \text{ rad} = 7.33 \text{ rad} \)

P5. A child rolls a ball on a level floor 3.5 m to another child. If the ball makes 15.0 revolutions, what is its diameter?

Solution:
With each revolution, the ball rolls along the floor distance equal to the circumference:
\( 2\pi r \)

\[
\text{number of revolutions} = \frac{\text{distance}}{\text{circumference}} = \frac{3.5 \text{ m}}{2\pi r} = 15.0
\]
\[
d = 2r = \frac{3.5 \text{ m}}{\pi(15.0)} = 7.4 \times 10^{-2} \text{ m}
\]

P7. (a) A grinding wheel 0.35 m in diameter rotates at 2500 rpm. Calculate its angular velocity in rad/s. (b) What are the linear speed and acceleration of a point on the edge of the grinding wheel?

Solution:
(a)
\[
1 \text{ min} \rightarrow 2500 \text{ revolutions} \rightarrow (2500 \text{ revolutions}) \frac{2\pi}{1 \text{ revolution}} \text{ radians}
\]
\[
1 \text{ sec} \rightarrow \frac{2500}{60} \text{ revolutions} \rightarrow \left( \frac{2500}{60} \text{ revolutions} \right) \frac{2\pi}{1 \text{ revolution}} \text{ radians}
\]
\[
\omega = \frac{\Delta \theta}{\Delta t} = \left( \frac{2500}{60} \text{ revolutions} \right) \frac{2\pi}{1 \text{ revolution}} \text{ radians} = 261.8 \text{ rad/s}
\]

(b)
\[
v = \omega r = (261.8 \text{ rad/s}) \left( \frac{0.35 \text{ m}}{2} \right) = 46 \text{ m/s}
\]

There is no tangential acceleration, only centripetal:
\[
a = \frac{v^2}{r} = \omega^2 r = 1.2 \times 10^4 \text{ m/s}^2
\]

P11. How fast (in rpm) must a centrifuge rotate if a particle 7.0 cm from the axis of rotation is to experience an acceleration of 100,000 g's?

Solution:
If the angular velocity is constant, there will be no tangential acceleration. There will be only centripetal acceleration (which is in the radial direction, towards the center):

\[ a_c = \frac{v^2}{r} = \omega^2 r = (100,000 \times 9.8 \text{ m/s}^2) = 9.8 \times 10^5 \text{ m/s}^2 \]

\[ \omega = \sqrt{\frac{9.8 \times 10^5 \text{ m/s}^2}{0.07 \text{ m}}} = 3741 \frac{\text{rad}}{\text{s}} = 3741 \frac{\text{rad} \cdot \text{revolutions}}{1 \text{ s} \cdot \left( \frac{\text{revolutions}}{2\pi \text{ rad}} \right)} = 3.6 \times 10^4 \text{ rpm} \]

**P21.** The tires of a car make 65 revolutions as the car reduces its speed uniformly from 95 km/h to 45 km/h. The tires have a diameter of 0.80 m (a) What was the angular acceleration of the tires? (b) If the car continues to decelerate at this rate, how much more time is required for it to stop?

**Solution:**
If the tires do not slip: the linear velocity along the edge of the tires is equal to the linear velocity of the center of the tires which is equal to the linear velocity of the car itself.

\[ v_1 = 95 \text{ km/h} = 26.39 \text{ m/s} \]
\[ v_2 = 45 \text{ km/h} = 12.5 \text{ m/s} \]

\[ (a) \]
\[ \omega^2 = \omega_1^2 + 2\alpha \Delta \theta \]
\[ \alpha = \frac{\omega^2 - \omega_1^2}{2 \Delta \theta} = \frac{v_1^2 - v_2^2}{r^2 2 \Delta \theta} = \frac{(12.5 \text{ m/s})^2 - (26.39 \text{ m/s})^2}{(0.40 \text{ m})^2 2 (65 \times 2\pi \text{ rad})} = -4.133 \frac{\text{rad}}{\text{s}^2} \]

\[ (b) \]
\[ \omega_3 = \omega_2 + \alpha t = \frac{v_2}{r} + \alpha t \]
\[ 0 \frac{\text{rad}}{\text{s}} = \frac{12.5 \text{ m/s}}{0.40 \text{ m}} - 4.133 \frac{\text{rad}}{\text{s}^2} (t) \Rightarrow t = 7.6 \text{ s} \]

**P33.** To get a flat, uniform cylindrical satellite spinning at the correct rate, engineers fire four tangential rockets as shown in the diagram. If the satellite has a mass of 3600 kg and a radius of 4.0 m, what is the required steady force of each rocket if the satellite is to reach 32 rpm in 5.0 min?

![End view of cylindrical satellite](image)

**Solution:**
All the forces will balance out → there will be no net force.
The torque of the forces, however, will not cancel out, but it will add up, because each force will cause rotation in the same direction.

\[ \tau = FR \]
\[ \tau_{net} = 4FR \]

The angular acceleration of the satellite will be:

\[ \alpha = \frac{\Delta \omega}{\Delta t} = \frac{(32 \text{ rad}) \left( \frac{32 \text{ rad}}{5.0 \text{ min}} \right) \left( \frac{1 \text{ min}}{60 \text{ sec}} \right) - 0 \text{ rad}}{5.0 \text{ min} \left( \frac{1 \text{ min}}{60 \text{ sec}} \right)} = 0.0117 \text{ rad/s}^2 \]

The torque is proportional to the angular acceleration. The coefficient of proportionality is the Moment of Inertia, which for a solid cylinder is equal to \( \frac{mR^2}{2} \).

\[ \tau_{net} = I \alpha = \left( \frac{mR^2}{2} \right) \alpha = \left( \frac{3000 \text{ kg}(4.0 \text{ m})^2}{2} \right) (0.0117 \text{ rad/s}^2) = 3.217 \times 10^2 \text{ N.m} \]

\[ F = \frac{\tau_{net}}{4R} = \frac{3.217 \times 10^2 \text{ N.m}}{4(4.0 \text{ m})} = 20.1 \text{ N} \]

**P37.** A centrifuge rotor rotating at 10,300 rpm is shut off and is eventually brought uniformly to rest by a frictional torque of 1.20 N · m. If the mass of the rotor is 4.80 kg and it can be approximated as a solid cylinder of radius 0.0710 m, through how many revolutions will the rotor turn before coming to rest, and how long will it take?

**Solution:**

\[ I = \frac{mR^2}{2} \rightarrow \alpha = \frac{\tau}{I} = \frac{-1.20 \text{ N.m}}{\frac{(4.80 \text{ kg})(0.0710 \text{ m})^2}{2}} = -99.187 \text{ rad/s}^2 \]

The acceleration is negative, because the rotor is being stopped, that is the torque is opposite to the direction of rotation.

\[ \omega_1 = (10,300 \text{ rev/min}) \left( \frac{1 \text{ min}}{60 \text{ sec}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 1.0786 \times 10^3 \text{ rad/s} \]

\[ \omega_2^2 = \omega_1^2 + 2\alpha \Delta \theta \]

\[ (0.0 \text{ rad/s})^2 = (1.0786 \times 10^3 \text{ rad/s})^2 + 2(-99.187 \text{ rad/s}^2)\Delta \theta \]

\[ \Delta \theta = 5.865 \times 10^3 \text{ rad} \rightarrow 5.865 \times 10^3 \text{ rad} \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 993 \text{ rev.} \]

\[ \omega_2 = \omega_1 + \alpha t \Rightarrow t = \frac{\Delta \omega}{\alpha} = \frac{-1.0786 \times 10^3 \text{ rad/s}}{-99.187 \text{ rad/s}^2} = 10.87 \text{ s} \]

**P43.** A centrifuge rotor has a moment of inertia of 3.75 \times 10^{-2} \text{ kg.m}^2. How much energy is required to bring it from rest to 8250 rpm?

**Solution:**

\[ KE_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2} (3.75 \times 10^{-2} \text{ kg.m}^2) \left( \frac{8250 \text{ rev}}{60 \text{ sec}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right)^2 = 1.40 \times 10^4 \text{ J} \]
**P44.** An automobile engine develops a torque of 280 N \cdot m at 3800 rpm. What is the power in watts and in horsepower?

**Solution:**

\[ W = F \cdot l = \frac{\tau}{r} \cdot l = \tau \cdot \Delta \theta \]

\[ P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \tau \frac{\Delta \theta}{\Delta t} = \tau \cdot \omega \]

\[ P = \tau \cdot \omega = (280 \text{ N.m})(3800 \text{ rev/min})(\frac{1 \text{ min}}{60 \text{ sec}})(\frac{2\pi \text{ rad}}{1 \text{ rev}}) = 1.114 \times 10^5 \text{ W} \]

\[ P = 1.114 \times 10^5 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 1.5 \times 10^2 \text{ hp} \]

**P45.** A bowling ball of mass 7.3 kg and radius 9.0 cm rolls without slipping down a lane at 3.3 m/s. Calculate its total kinetic energy.

**Solution:**

When there is no slipping → the translational velocity of the ball is equal to the translation velocity of the points on the edge of the ball → \( \omega = \frac{v}{R} \)

\[ KE_{tot} = KE_{trans} + KE_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2} \frac{mR^2}{5} \frac{v^2}{R^2} \]

\[ KE_{tot} = 56 \text{ J} \]

**P47.** A merry-go-round has a mass of 1640 kg and a radius of 7.50 m. How much net work is required to accelerate it from rest to a rotation rate of 1.00 revolution per 8.00 s? Assume it is a solid cylinder.

**Solution:**

\[ \omega_1 = 0 \text{ rad/s} \]

\[ \omega_2 = \frac{1.00 \text{ rev}}{8.00 \text{ s}} \cdot \frac{\frac{2\pi \text{ rad}}{1 \text{ rev}}}{1 \text{ rev}} = 0.7854 \text{ rad/s} \]

\[ \Delta E = W_{net} \quad \text{(work done by non-conservative forces)} \]

\[ \Delta KE = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2 = \frac{1}{2} \frac{mR^2}{5} \omega_2^2 - 0 = \frac{1}{4} (1640 \text{ kg})(7.50 \text{ m})^2 (0.7854 \text{ rad/s})^2 \]

\[ \Delta KE = 1.42 \times 10^4 \text{ J} \]

\[ W_{net} = 1.42 \times 10^4 \text{ J} \]

**P54.** A diver can reduce her moment of inertial by a factor of about 3.5 when changing from the straight position to the tuck position. If she makes 2.0 rotations in 1.5s when in the tuck position, what is her angular speed (rev/s) when in the straight position?

**Solution:**

There is no net torque → The angular momentum is conserved

\[ L_1 = L_2 \]

\[ I_1 \omega_1 = I_2 \omega_2 \]
An asteroid of mass $1.0 \times 10^5$ kg, traveling at a speed of 30 km/s relative to the Earth, hits the Earth at the equator tangentially, and in the direction of Earth's rotation. Use angular momentum to estimate the percent change in the angular momentum to estimate the percent change in the angular speed of the Earth as a result of the collision.

**Solution:**
The angular momentum is conserved. 

$$\omega_{ia} = \frac{v_i}{R_e} = 4.7 \times 10^{-3} \text{ rad/s} \quad R_e = \text{Earth's radius}$$

$$\omega_{ie} = \frac{2\pi}{24(3600 \text{ s})} = 7.27 \times 10^{-5} \text{ rad/s}$$

$$I_e \omega_{ie} + I_o \omega_{ia} = (I_e + I_o) \omega_f$$

$$\frac{\omega_f - \omega_{ia}}{\omega_{ie}} = \frac{\frac{I_e \omega_{ie} + I_o \omega_{ia} - \omega_{ie}}{(I_e + I_o) \omega_{ie}}}{\omega_{ie}} = \frac{I_o (\omega_{ia} - \omega_{ie})}{(I_e + I_o) \omega_{ie}}$$

$$I_o = m_o R_e^2$$

$$I_E = \frac{2}{5} M_e R_e^2$$

$$\frac{\omega_f - \omega_{ie}}{\omega_{ie}} = \frac{m_o \frac{2 \pi}{M_e} - \omega_{ie}}{\left(\frac{2}{5} M_e + m_o \frac{2 \pi}{M_e}\right) \omega_{ie}} = \frac{m_o \frac{2 \pi}{M_e} - \omega_{ie}}{\left(\frac{2}{5} M_e + m_o \frac{2 \pi}{M_e}\right) \omega_{ie}} = 2.7 \times 10^{-16} \%$$