Graphing Notes

• Why use graphs?
  • Illustrates the relationship between two variables.
  • Leaves out other potential factors that may influence those variables.
  • NOTE: A variable is a symbol or name that stands for a value. The value is therefore flexible and can represent a variety of specific data.
    • For example, I could use the symbol Y to represent the price of toothpaste. Obviously, toothpaste can take on any number of prices.

• Relationship between tables and graphs.
  • A graph is simply a picture of the numerical relationships exhibited in a table.

• Vertical axis is referred to as the Y-axis.
• Horizontal axis is referred to as the X-axis.
• Origin – where the two axis cross.

Example 1 – graphing a simple linear relationship (a straight line)

<table>
<thead>
<tr>
<th>Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Equation: \( y = a + bx \) is the generalized form for a linear equation.
y – the dependent variable
a – vertical intercept
b – slope  
\( x \) – independent variable

The specific form for this equation is:  
\[ y = 0 + 3x \quad \text{OR} \quad y = 3x \]

- **Details**: tables show numeric links that can then be shown in a graph or represented by an equation. Likewise, you could begin with either an equation or a graph and work out the other two from there.

**Example 2** – starting with an equation  
\[ y = 2 + \frac{1}{2}x \quad – 	ext{create a table by plugging in various values for} \ x \text{ and seeing what the resulting} \ y \text{ is.} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 ½</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3 ½</td>
</tr>
</tbody>
</table>

– this can then be graphed

- **Positive relationships/slopes versus negative relationships/slopes.**
  - Positive: an increase in one variable increases the other variable. Also, a decrease in one variable decreases the other variable.
  - Negative: an increase in one variable decreases the other variable. Also, a decrease in one variable increases the other variable.

- **More details:**
  - We call \( y \) the dependent variable, because its value depends on \( x \).
  - \( x \) is called the independent variable.
  - Usually the independent variable is placed on the horizontal axis and the dependent variable is placed on the vertical axis. However, economists aren’t consistent about this and it will be necessary to learn the standard form for each type of graph. I will point this out as we go along. [Note that even when the
dependent variable is placed on the horizontal axis, the slope is still calculated as rise/run.]

- Sometimes, it is not necessarily clear that there is a causal relationship between two variables, although the term dependent variable implies that there is such a relationship. This is the difference between correlation and causation.

- The slope quantifies how much an increase (decrease) in one variable increases (decreases) another variable.

- The simplest way to calculate slope is:

\[
Slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{\text{first y-coordinate} - \text{second y-coordinate}}{\text{first x-coordinate} - \text{second x-coordinate}}
\]

- Just make certain you always start from the same point (i.e. use the same point to determine the first coordinate of both x and y.

- The slope in Example 1 above is therefore (using the first two combinations listed in the table):

\[
Slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{3 - 6}{1 - 2} = \frac{-3}{-1} = 3
\]

- This is a positive slope, meaning that both x and y change in the same direction: when x goes up, y goes up and when x goes down, y goes down. This is also called a direct relationship.
• It is also possible to have a negative slope or an indirect relationship between x and y. For example, consider the points (1, 2) and (2, 1).
  
  \[ \text{Slope} = -1/1 = -1 \]

• The slope of a line is the marginal value of y due to a change in x. For those of you who have had some calculus, the first derivative of an equation will give you the marginal value.

• When analyzing a particular relationship between two variables, we hold other factors constant – ceteris paribus (for a definition, see page 17).
  • If one of the factors underlying the relationship between two variable changes, the curve reflecting that relationship will shift.
  • Example from book (page 23) – a uniform increase in grading standards.
  • Example – an increase in income will shift a demand curve (which reflect a relationship between price and quantity demanded).

• Shifts can be parallel (i.e. maintain the same slope while changing position). See Figure A.2.
• Shifts can also change both slope and position. See Figure A.3.

• Relationships between two variables can be nonlinear as well. In this case, the slope changes throughout the relationship (i.e. the slope is different at each point on the curve).
  • NOTE: A linear curve means a straight line.
  • The slope of a nonlinear curve at a particular point is equal to the slope of a straight line just tangent (touching) the curve at that point. (NOTE: calculus can simplify this process tremendously.)
  • For a non-linear curve, the slope at a particular point can be found by taking the derivative and plugging in the relevant x value. For example, the derivative of \( y = x^2 + 2x + 3 \) is \( 2x + 2 \). To find the slope at point (1, 6), plug 1 into the derivative equation to get 4. The slope at that point is 4. Another way of finding the slope at a particular point on a non-linear curve is to find the slope of the line that is tangent to it (just touches that point).
Example 3 – $y = 1/x$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Undefined and so approaching infinity</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

How would we calculate the slope at (1,1)?
1. Using calculus, it is $1/x^2 = -1/1^2 = -1$.
2. To calculate the slope at x=1, y=1 (the point where our straight line touches the above curve), we calculate the slope of that line:

$$Slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{2 - 0}{0 - 2} = \frac{2}{-2} = -1$$

So, both methods get us the same result!
• **Infinite slope**: no matter what happens to the price of air, the same quantity is supplied.

![Graph of Infinite Slope](image)

• **Zero slope**: the price of eggs is completely unrelated to the quantity supplied. This situation typifies perfect competition, which we will discuss later in the semester.

![Graph of Zero Slope](image)

• The units used matter when calculating slope.
  • Consider the difference in slope when price is measured in dollars versus the same price in pennies.
  • This detail will matter when we discuss elasticities.

• **Abstract graphs**: it is time consuming to always use specific numbers in graphs, so it is important that you become comfortable with the overall relationship between two variables without the necessity of specific numbers.

• Just because a relationship exists between two variables, does not mean that one *causes* the other. It is therefore important to have a theory to support any hypothesized relationship.
  • Example – sunspots and falls in the stock market.
  • Example – students who are smarter study more (see pages 24-25).
• For those of you who would like some additional practice. I recommend you follow the book’s advice and check out the following link:

http://syllabus.syr.edu/cid/graph/TOCbook.html